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Abstract. This article studies a development of SEIR standard model for the spread of hepatitis B virus. The model is developed by considering immunized and carrier compartments of population. The model includes immunized, suspected, exposed, infected, carrier, and recovered compartments and written as MSEICR. Some of new born and the suspected are given vaccines and the infected is given a treatment. These strategies aim to reduce transmissions of hepatitis B virus in the population. The existence and stability of endemic and non-endemic equilibrium points are analysed via basic reproduction number ($\mathcal{R}_0$) which is derived from the next generation matrix method. The results showed that the endemic equilibrium point does not exist when $\mathcal{R}_0 < 1$. The endemic will appear when the value of $\mathcal{R}_0 > 1$. Sensitivity analyses showed that vaccination and treatment may reduce the spread of hepatitis B virus and also eliminate endemic condition. Some simulations were conducted to visualize the effects of vaccination and treatment on the existence and stability of endemic equilibrium point.

1. Introduction

Hepatitis is an inflammation of the liver. An inflamed liver can develop into fibrosis, cirrhosis or liver cancer. There are at least five types of hepatitis, namely type A, B, C, D, and E. Each type of hepatitis can cause death and has the potential to become an endemic disease. Infections of hepatitis B and C cause chronic liver disease to the many of people. These infections are the main causes of liver cirrhosis and liver cancer. Epidemics of hepatitis B and C viruses cause millions of people to die and have spread more widely. There are many people who die due to liver disease caused by viral hepatitis, liver failure, and liver cancer.

The dynamics of the spread of hepatitis in humans from an infected human to another human who are susceptible to the disease can be expressed in the form of mathematical modelling. The SEIR model is very often used to study the transmission of hepatitis virus. Mathematical modelling in transmission of diseases used standard model, SIS, SIR, and SEIR. These models have been developed by adding some compartments for examples carriers, immunized, migration, and the like [1, 2, 3, 4, 5]. Some strategies on the spread model of hepatitis B virus have been imposed in order to reduce and eliminate the disease, for examples vaccination, inject drugs, treatments, and control the cost associated with the applied strategies [6, 7, 8, 9, 10]

Vaccination in the infants can give an effect as an effort to prevent from hepatitis B. One effort to prevent effectively from hepatitis B is to inject hepatitis B vaccines. Vaccination is given 3 times, namely

when new children, children aged 1 month, and 3-6 months. Besides providing vaccines for the infants, there are also several preventive and treatment as the effort to reduce the spread of hepatitis B virus. Model of spread of hepatitis B has been studied to investigate characteristics the spread of hepatitis B virus. This study analysed the effect of migrant people on the host country of the spread of hepatitis and added transmission migrant class and acute class [11].

The studies on the model of spread of hepatitis B virus in [12] considered vaccination and treatment as controls. The model included suspected, exposed, infected, carriers, and recovered compartments. The studies focused on analyse the existence and stability of disease-free and endemic equilibrium points of the model. The strategies to minimize the infected and the related cost were formulated. Spread model of hepatitis B virus in [13] also considered the impacts of treatment. The treatment as a control was analysed to investigate the parameter which gives an effect on the dynamics. Benefit and sensitivity of this strategy were also analysed analytically and via simulations.

The dynamics of spread of hepatitis B virus included transmissions of hepatitis B virus was formulated by considering different classification individual from the population [14]. The role of vaccination and treatment in the infected individuals to control the transmission in compartments was considered in the standard SEIR model and also imposed immunized compartment into the model. The model was improved and becomes an MSEIR model.

In this article, the dynamics of spread of hepatitis B virus in the form of MSEIR model and SEICR model were improved by considering both immunized and carriers into the model. The improved model becomes an MSEICR, which included immunized, suspected, infected, carriers, and recovered compartments. Vaccination and treatment were included in the mechanics of the model. This model focused on determination of basic reproduction number and simulation to confirm the analytical result. Sensitivity analysis was conducted to investigate the effect of vaccination and treatment as the efforts to prevent the spread of hepatitis B virus.

2. Development of SEIR model on spread of hepatitis B virus

The dynamics of spread model on hepatitis B virus is commonly divided into four compartments. The compartments are susceptible (S), exposed (E), infected (I), and recovered (R), see for example [15]. In the spread of hepatitis B virus, compartments maybe considered in six compartments by adding immunized (M) and carriers (C) compartments. The immunized new born is classified as an immunized compartment when the infant is given vaccines. The carrier compartment is a group of people who acts as carrying hepatitis B virus. In this developing of the model, the total population is defined as $\boldsymbol{N(t) = M(t) + S(t) + E(t) + I(t) + C(t) + R(t)}$. Total population is assumed to be constant in time. The diagram of spread of hepatitis B virus with vaccination and treatment is given in figure 1.

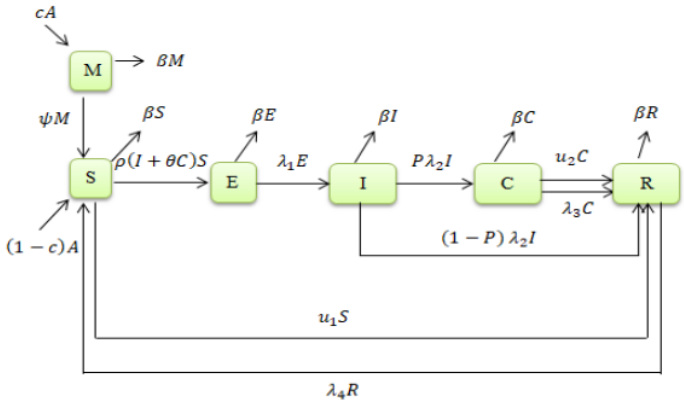

Figure 1. The flow diagram of spread hepatitis B virus with vaccination and treatment.

There are several assumptions to be used in the MSEICR model. The immunized of new born with a constant rate of cA is classified as immunized compartment. The new born who are not immunized at the rate of $(1-c)A$ and who have exhausted the effects of immunization at the rate of ψM are classified as suspected compartment. Individuals in suspected compartment if giving vaccines with a rate of $u_1 S$ will move to recovered compartment. Individuals who are in the suspected interacts with the infected individuals and carriers. The suspected individual will move to the exposed compartment with a rate of $\rho(I+\theta C)S$. Individuals who have been exposed will move to the infected with a rate of $\lambda_1 E$. The acute infected individuals can move to the carriers with a rate of $P\lambda_2 I$, and while the acute infected will move from the infected compartment to the recovered compartment with a rate of $(1-P)\lambda_2 I$. Because of treatment and natural recovery, the carriers will move to the recovered with the rates of $u_2 C$ and $\lambda_3 C$. Because of loss of immunity, individuals from the recovered will move to the suspected with a rate of $\lambda_4 R$. Each compartment has its own rate of natural death.

From figure 1, the dynamics of change of each compartment which are stated in the form of differential equations system are as follow

$$\begin{aligned}
\frac{dM(t)}{dt} &= cA - \beta M - \psi M \\
\frac{dS(t)}{dt} &= (1-c)A + \psi M - \beta S - \rho(I+\theta C)S - u_1 S + \lambda_4 R \\
\frac{dE(t)}{dt} &= \rho(I+\theta C)S - \lambda_1 E - \beta E \\
\frac{dI(t)}{dt} &= \lambda_1 E - P\lambda_2 I - \beta I - (1-P)\lambda_2 I \\
\frac{dC(t)}{dt} &= P\lambda_2 I - \beta C - u_2 C - \lambda_3 C \\
\frac{dR(t)}{dt} &= u_2 C + \lambda_3 C + (1-P)\lambda_2 I + u_1 S - \lambda_4 R - \beta R \,.
\end{aligned} \tag{1}$$

By assumption that the total population is constant, then model (1) can be expressed in a simpler term in the form of proportion between sub population and the total population. Suppose that we write new variables

$$x_1(t) = \frac{M(t)}{N(t)}, x_2(t) = \frac{S(t)}{N(t)}, x_3(t) = \frac{E(t)}{N(t)}, x_4(t) = \frac{I(t)}{N(t)}, x_5(t) = \frac{C(t)}{N(t)}, \text{ and } x_6(t) = \frac{R(t)}{N(t)}.$$

After doing differentiation and simplification, model (1) is then written as

$$\begin{aligned}
\frac{dx_1}{dt} &= c\beta - \beta x_1 - \psi x_1 \\
\frac{dx_2}{dt} &= (1-c)\beta + \psi x_1 - \beta x_2 - \rho(x_4 + \theta x_5)x_2 - u_1 x_2 + \lambda_4 x_6 \\
\frac{dx_3}{dt} &= \rho(x_4 + \theta x_5)x_2 - \lambda_1 x_3 - \beta x_3 \\
\frac{dx_4}{dt} &= \lambda_1 x_3 - \beta x_4 - \lambda_2 x_4 \\
\frac{dx_5}{dt} &= P\lambda_2 x_4 - (\beta + u_2 + \lambda_3)x_5 \\
\frac{dx_6}{dt} &= (u_2 + \lambda_3)x_5 + (1-P)\lambda_2 x_4 + u_1 x_2 - (\lambda_4 + \beta)x_6 \,.
\end{aligned} \tag{2}$$

The endemic and non-endemic equilibrium points are found by solving the simple isoclines of model (2) simultaneously, i.e. $\frac{dx_1}{dt} = 0, \frac{dx_2}{dt} = 0, \frac{dx_3}{dt} = 0, \frac{dx_4}{dt} = 0, \frac{dx_5}{dt} = 0$, and $\frac{dx_6}{dt} = 0$.
Then we have a non-endemic equilibrium point

$$\begin{aligned}
T_0 &= \left(x_{1_0}, x_{2_0}, x_{3_0}, x_{4_0}, x_{5_0}, x_{6_0}\right) \\
&= \left(\frac{c\beta}{\beta+\psi}, \frac{\lambda_4\psi - c\beta\lambda_4 + \lambda_4\beta + \psi\beta - c\beta^2 + \beta^2}{\beta^2 + \lambda_4\psi + \psi u_1 + \lambda_4\beta + \psi\beta + \beta u_1}, 0,0,0, \frac{u_1(\beta - c\beta + \psi)}{(\lambda_4\beta + \psi\lambda_4 + \beta^2 + \beta u_1 + \beta\psi + \psi u_1)}\right).
\end{aligned}$$

There exists an endemic equilibrium point written as $T_1 = \left(x_{1_1}, x_{2_1}, x_{3_1}, x_{4_1}, x_{5_1}, x_{6_1}\right)$, where $T_1 \in R_+^6$. Claim of the existence of $T_1 \in R_+^6$ will be proven via basic reproduction number which is explained in the next section.

3. Basic reproduction number ($\mathcal{R}_0$) of the model

Basic reproduction number ($\mathcal{R}_0$) is used to determine the level of the spread of the disease. The basic reproduction number can be determined using next generation matrix [16]. Following the methods of calculating $\mathcal{R}_0$, we let $\mathcal{F}_i(x)$ defines the rate of adding new infections to the compartment i and $\mathcal{V}_i(x)$ defines the rate of individual displacement in the compartment i, then $\mathcal{F}_i(x)$ and $\mathcal{V}_i(x)$ from model (2) are written as follow

$$\mathcal{F}(x) = \begin{pmatrix} \rho(x_4 + \theta x_5)x_2 \\ 0 \\ 0 \end{pmatrix} \tag{3}$$

and

$$\mathcal{V}(x) = \begin{pmatrix} (\lambda_1 + \beta)x_3 \\ -\lambda_1 x_3 + (\beta + \lambda_2)x_4 \\ -P\lambda_2 x_4 + (\beta + u_2 + \lambda_3)x_5 \end{pmatrix}. \tag{4}$$

From equations (3) and (4) we have matrices F and V as

$$F = \frac{\partial \mathcal{F}_i(T_0)}{\partial x_j} = \begin{pmatrix} 0 & \rho x_2 & \rho\theta x_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{5}$$

and

$$V = \frac{\partial \mathcal{V}_i(T_0)}{\partial x_j} = \begin{pmatrix} c_1 & 0 & 0 \\ -\lambda_1 & c_2 & 0 \\ 0 & -P\lambda_2 & c_3 \end{pmatrix}, \tag{6}$$

where $c_1 = \lambda_1 + \beta$, $c_2 = \beta + \lambda_2$, $c_3 = \beta + u_2 + \lambda_3$, and $x = (x_3, x_4, x_5)$. Inverse of matrix V is

$$V^{-1}(x) = \begin{pmatrix} \frac{1}{c_1} & 0 & 0 \\ \frac{\lambda_1}{c_1 c_2} & \frac{1}{c_2} & 0 \\ \frac{\lambda_1 P\lambda_2}{c_1 c_2 c_3} & \frac{P\lambda_2}{c_2 c_3} & \frac{1}{c_3} \end{pmatrix}. \tag{7}$$

The basic reproduction number ($\mathcal{R}_0$) is calculated using matrices F and V^{-1}. Then, from equations (5) and (7) we have

$$FV^{-1} = \begin{pmatrix} 0 & \rho x_2 & \rho\theta x_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{c_1} & 0 & 0 \\ \frac{\lambda_1}{c_1 c_2} & \frac{1}{c_2} & 0 \\ \frac{\lambda_1 P\lambda_2}{c_1 c_2 c_3} & \frac{P\lambda_2}{c_2 c_3} & \frac{1}{c_3} \end{pmatrix} = \begin{pmatrix} \frac{\rho x_2 \lambda_1}{c_1 c_2} + \frac{\rho\theta x_2(\lambda_1 P\lambda_2)}{c_1 c_2 c_3} & \frac{\rho x_2}{c_2} + \frac{\rho\theta x_2(P\lambda_2)}{c_2 c_3} & \frac{\rho\theta x_2}{c_3} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{8}$$

Therefore, the eigenvalues of matrix FV^{-1} is found by solving the following equation

$$\det(F(x)V^{-1}(x) - rI) = \det \begin{pmatrix} \frac{\rho x_2 \lambda_1}{c_1 c_2} + \frac{\rho\theta x_2(\lambda_1 P\lambda_2)}{c_1 c_2 c_3} - r & \frac{\rho x_2}{c_2} + \frac{\rho\theta x_2(P\lambda_2)}{c_2 c_3} & \frac{\rho\theta x_2}{c_3} \\ 0 & -r & 0 \\ 0 & 0 & -r \end{pmatrix} = 0 .$$

From the last equation we have the characteristic equation $f(r) = \left(\frac{\rho x_2 \lambda_1}{c_1 c_2} + \frac{\rho\theta x_2(\lambda_1 P\lambda_2)}{c_1 c_2 c_3} - r\right) r^2 = 0$. The eigenvalues are $r_1 = \frac{\rho x_2 \lambda_1}{c_1 c_2} + \frac{\rho\theta x_2(\lambda_1 P\lambda_2)}{c_1 c_2 c_3}$ and $r_{2,3} = 0$. Spectral radius of the next generation matrix FV^{-1} is $\frac{\rho x_2 \lambda_1}{c_1 c_2} + \frac{\rho\theta x_2(\lambda_1 P\lambda_2)}{c_1 c_2 c_3}$ which is written as $\mathcal{R}_0 = \frac{\rho x_2 \lambda_1}{c_1 c_2} + \frac{\rho\theta x_2(\lambda_1 P\lambda_2)}{c_1 c_2 c_3}$. After substituting the value of c_1, c_2, c_3, and x_2 as a component of the equilibrium point T_0, the basic reproduction number is written as

$$\mathcal{R}_0 = \frac{\rho\lambda_1(\lambda_4\psi - c\beta\lambda_4 + \lambda_4\beta + \psi\beta - c\beta^2 + \beta^2)}{(\lambda_1+\beta)(\beta+\lambda_2)(\beta^2+\lambda_4\psi+\psi u_1+\lambda_4\beta+\psi\beta+\beta u_1)} + \frac{\rho\theta(\lambda_1 P\lambda_2)(\lambda_4\psi - c\beta\lambda_4 + \lambda_4\beta + \psi\beta - c\beta^2 + \beta^2)}{(\lambda_1+\beta)(\beta+\lambda_2)(\beta+u_2+\lambda_3)(\beta^2+\lambda_4\psi+\psi u_1+\lambda_4\beta+\psi\beta+\beta u_1)} . \tag{9}$$

The value of $\mathcal{R}_0$ measures whether the disease will spread and become endemic or will disappear from the population. When $\mathcal{R}_0 < 1$, the disease will disappear and the exposed, infected, and carriers compartments tend to zero as time goes on. While when $\mathcal{R}_0 > 1$, the disease will spread and become endemic. This means that each compartments will be positive valued for a long time. In another words, in the case of $\mathcal{R}_0 > 1$ the endemic equilibrium point T_1 exists and stable.

One effort to prevent the increase of infected individuals with hepatitis B virus is to implement the strategy to prevent the disease transmission. Preventing the spread of hepatitis B virus is a strategy to avoid increasing the number of exposed, infected, and carrier individuals. Hepatitis B virus will not be endemic in the population when $\mathcal{R}_0 < 1$. Giving vaccines for the infant and suspected compartments is one strategy to reduce the spread of hepatitis B virus. Treatment in the infected compartment is also a strategy to reduce the number of infected individual and may also reduce the transmission of hepatitis B virus to the suspected compartment.

4. Sensitivity analysis of $\mathcal{R}_0$

The basic reproduction number $\mathcal{R}_0$ determines the existence of endemic equilibrium point T_1. From the equation (9), the value of $\mathcal{R}_0$ depends on some parameters of model (2). Here, we focus on the parameter which are related to the giving vaccines for immunized and suspected individuals and also to treatment for infected individual. Sensitivity for $\mathcal{R}_0$ relates to changes of parameter $c\beta$ (rate of immunized for the infant), u_1 (proportion of individuals from suspected compartment to recovered compartment with vaccination), u_2 (proportion of individual from carriers to the recovered compartment with treatment), and P (proportion of individual from infected compartment to the carriers).

The change of $\mathcal{R}_0$ compared with the change of $c\beta$ is analysed from derivative of $\mathcal{R}_0$ with respect to $c\beta$ as follows.

$$\frac{\partial \mathcal{R}_0}{\partial (c\beta)} = \left(-\frac{\rho\lambda_1(\lambda_4+2c\beta)}{c_1c_2(\beta^2+\lambda_4\psi+\psi u_1+\lambda_4\beta+\psi\beta+\beta u_1)} - \frac{\rho\theta(\lambda_1 P\lambda_2)(\lambda_4+2c\beta)}{c_1c_2c_3(\beta^2+\lambda_4\psi+\psi u_1+\lambda_4\beta+\psi\beta+\beta u_1)} \right)$$
$$\left(\frac{c\beta c_1c_2(\beta^2+\lambda_4\psi+\psi u_1+\lambda_4\beta+\psi\beta+\beta u_1)}{\rho\lambda_1(\lambda_4\psi-c\beta\lambda_4+\lambda_4\beta+\psi\beta-c\beta^2+\beta^2)} + \frac{c\beta c_1c_2c_3(\beta^2+\lambda_4\psi+\psi u_1+\lambda_4\beta+\psi\beta+\beta u_1)}{\rho\theta(\lambda_1 P\lambda_2)(\lambda_4\psi-c\beta\lambda_4+\lambda_4\beta+\psi\beta-c\beta^2+\beta^2)} \right)$$
$$= -\frac{c\beta(\lambda_4+2c\beta)}{(\lambda_4\psi-c\beta\lambda_4+\lambda_4\beta+\psi\beta-c\beta^2+\beta^2)} - \frac{c\beta(\lambda_4+2c\beta)}{P\lambda_2(\lambda_4\psi-c\beta\lambda_4+\lambda_4\beta+\psi\beta-c\beta^2+\beta^2)}$$
$$-\frac{\theta P\lambda_2(\lambda_4+2c\beta)c\beta}{c_3(\lambda_4\psi-c\beta\lambda_4+\lambda_4\beta+\psi\beta-c\beta^2+\beta^2)} - \frac{c\beta(\lambda_4+2c\beta)}{(\lambda_4\psi-c\beta\lambda_4+\lambda_4\beta+\psi\beta-c\beta^2+\beta^2)}$$
$$= -\frac{c\beta(\lambda_4+2c\beta)}{(\lambda_4\psi-c\beta\lambda_4+\lambda_4\beta+\psi\beta-c\beta^2+\beta^2)}\left(2+\frac{1}{P\lambda_2}+\frac{\theta P\lambda_2}{c_3}\right) < 0\,.$$

Hepatitis B vaccine is a vaccine used to prevent liver infections due to the hepatitis B virus. This vaccine works by stimulating the immune system to produce antibodies that can fight against viruses. Thus, vaccination gives consequences that the suspected individual can move and become recovered individual. The impact of vaccination hepatitis B virus can be analysed via derivative of $\mathcal{R}_0$ with respect to u_1 as follow.

$$\frac{\partial \mathcal{R}_0}{\partial u_1} = \left(-\frac{\rho\lambda_1(\lambda_4\psi-c\beta\lambda_4+\lambda_4\beta+\psi\beta-c\beta^2+\beta^2)(\psi+\beta)}{c_1c_2(\beta^2+\lambda_4\psi+\psi u_1+\lambda_4\beta+\psi\beta+\beta u_1)^2} - \frac{\rho\theta(\lambda_1 P\lambda_2)(\lambda_4\psi-c\beta\lambda_4+\lambda_4\beta+\psi\beta-c\beta^2+\beta^2)(\psi+\beta)}{c_1c_2c_3(\beta^2+\lambda_4\psi+\psi u_1+\lambda_4\beta+\psi\beta+\beta u_1)^2} \right)$$
$$\left(\frac{c_1c_2(\beta^2+\lambda_4\psi+\psi u_1+\lambda_4\beta+\psi\beta+\beta u_1)u_1}{\rho\lambda_1(\lambda_4\psi-c\beta\lambda_4+\lambda_4\beta+\psi\beta-c\beta^2+\beta^2)} + \frac{c_1c_2c_3(\beta^2+\lambda_4\psi+\psi u_1+\lambda_4\beta+\psi\beta+\beta u_1)u_1}{\rho\theta(\lambda_1 P\lambda_2)(\lambda_4\psi-c\beta\lambda_4+\lambda_4\beta+\psi\beta-c\beta^2+\beta^2)} \right)$$
$$= -\frac{u_1(\psi+\beta)}{(\beta^2+\lambda_4\psi+\psi u_1+\lambda_4\beta+\psi\beta+\beta u_1)} - \frac{u_1c_3(\psi+\beta)}{\theta P\lambda_2(\beta^2+\lambda_4\psi+\psi u_1+\lambda_4\beta+\psi\beta+\beta u_1)}$$
$$-\frac{\theta P\lambda_2(\psi+\beta)u_1}{c_3(\beta^2+\lambda_4\psi+\psi u_1+\lambda_4\beta+\psi\beta+\beta u_1)} - \frac{u_1(\psi+\beta)}{(\beta^2+\lambda_4\psi+\psi u_1+\lambda_4\beta+\psi\beta+\beta u_1)}$$
$$= -\frac{u_1(\psi+\beta)}{(\beta^2+\lambda_4\psi+\psi u_1+\lambda_4\beta+\psi\beta+\beta u_1)}\left(2+\frac{c_3}{\theta P\lambda_2}+\frac{\theta P\lambda_2}{c_3}\right) < 0.$$

Treatment of hepatitis B is to reduce symptoms, such as giving painkillers and maintaining the patient comfort and nutritional balance. Treatment of chronic hepatitis B virus depends on the severity of liver infection. Use of drugs serve to inhibit viral production and prevent liver damage. The effect of treatment of hepatitis B virus can be analysed via derivative of $\mathcal{R}_0$ with respect to u_2 as follow.

$$\frac{\partial \mathcal{R}_0}{\partial u_2} = \left(-\frac{\rho\theta(\lambda_1 P\lambda_2)(\lambda_4\psi - c\beta\lambda_4 + \lambda_4\beta + \psi\beta - c\beta^2 + \beta^2)}{c_1c_2{c_3}^2(\beta^2 + \lambda_4\psi + \psi u_1 + \lambda_4\beta + \psi\beta + \beta u_1)}\right)$$
$$\left(\frac{u_2c_1c_2(\beta^2 + \lambda_4\psi + \psi u_1 + \lambda_4\beta + \psi\beta + \beta u_1)}{\rho\lambda_1(\lambda_4\psi - c\beta\lambda_4 + \lambda_4\beta + \psi\beta - c\beta^2 + \beta^2)} + \frac{u_2c_1c_2c_3(\beta^2 + \lambda_4\psi + \psi u_1 + \lambda_4\beta + \psi\beta + \beta u_1)}{\rho\theta(\lambda_1 P\lambda_2)(\lambda_4\psi - c\beta\lambda_4 + \lambda_4\beta + \psi\beta - c\beta^2 + \beta^2)}\right)$$
$$= -\frac{\theta P\lambda_2 u_2}{{c_3}^2} - \frac{u_2}{c_3} < 0\,.$$

Based on the history of hepatitis, hepatitis virus can be divided into two parts, acute hepatitis and chronic hepatitis. Some acute hepatitis can be recovered completely, but some acute become carriers. In case of unchecked and uncontrolled, the acute hepatitis can develop into chronic liver disease. The effect of proportion of infected individual become carriers can be analysed via derivative $\mathcal{R}_0$ with respect to P as follow.

$$\frac{\partial \mathcal{R}_0}{\partial P} = \left(\frac{\rho\theta\lambda_1\lambda_2(\lambda_4\psi - c\beta\lambda_4 + \lambda_4\beta + \psi\beta - c\beta^2 + \beta^2)}{c_1c_2c_3(\beta^2 + \lambda_4\psi + \psi u_1 + \lambda_4\beta + \psi\beta + \beta u_1)}\right)$$
$$\left(\frac{Pc_1c_2(\beta^2 + \lambda_4\psi + \psi u_1 + \lambda_4\beta + \psi\beta + \beta u_1)}{\rho\lambda_1(\lambda_4\psi - c\beta\lambda_4 + \lambda_4\beta + \psi\beta - c\beta^2 + \beta^2)} + \frac{Pc_1c_2c_3(\beta^2 + \lambda_4\psi + \psi u_1 + \lambda_4\beta + \psi\beta + \beta u_1)}{\rho\theta(\lambda_1 P\lambda_2)(\lambda_4\psi - c\beta\lambda_4 + \lambda_4\beta + \psi\beta - c\beta^2 + \beta^2)}\right)$$
$$= \frac{P\theta\lambda_2}{c_3} + 1 > 0\,.$$

From the sensitivity analysis we found that $\frac{\partial \mathcal{R}_0}{\partial(c\beta)} < 0$, $\frac{\partial \mathcal{R}_0}{\partial u_1} < 0$, $\frac{\partial \mathcal{R}_0}{\partial u_2} < 0$, and $\frac{\partial \mathcal{R}_0}{\partial P} > 0$. In case of giving vaccines to the immunized infant, the value of $\mathcal{R}_0$ will be reduced as if increasing the vaccination value. Increasing the value of vaccination may reduce the value of $\mathcal{R}_0$ become less than one, which means that the hepatitis B virus disappear for the population. The effect of giving vaccination to the suspected and carriers may also lead the value of $\mathcal{R}_0$ become less than one. While the increasing value of P, portion from infected to the carriers, will increase the value of $\mathcal{R}_0$.

5. Numerical simulations

In this section, simulations will be carried out to show and investigate the sensitivity of immunization to infant, vaccination, and treatment of the spread of hepatitis B virus to the values of $\mathcal{R}_0$. The values of parameter which will be used in the simulations refers to table 1.

Table 1. Values of parameter related to the model of spread hepatitis B virus.

Parameters	Values	References
c	0.1	Assumed
β	0.0121	[16]
ρ	$0.8 - 20.49$	[16]
ψ	0.0015	Assumed
θ	$0 - 1$	[16]
λ_1	6 per year	[16]
λ_2	4 per year	[16]
λ_3	0.025 per year	[16]
λ_4	$0.03 - 0.06$	[16]
P	$0.05 - 0.9$	[16]

u_1	0.11	Assumed
u_2	0.1	Assumed

5.1. Effect of immunisation to the value of $\mathcal{R}_0$ and the equilibrium points

In this simulation, we suppose that the values of parameter of model (2) are given as $\beta = 0.0121$, $\rho = 7.5$, $\psi = 0.0015$, $\lambda_1 = 6$, $\lambda_2 = 4$, $\lambda_3 = 0.025$, $\lambda_4 = 0.05$, $P = 0.08, u_1 = 0.11$, and $u_2 = 0.1$ in appropriate units. There are five values of c used in this simulation, namely $c = 0.01$, $c = 0.05$, $c = 0.10$, $c = 0.40$, and $c = 0.50$. The effect of increasing value of c to the value of $\mathcal{R}_0$ and equilibrium point is given in table 2.

Table 2. The effect of vaccines on infant individual to the spread of hepatitis B virus.

Parameter c	**$\mathcal{R}_0$**	**Equilibrium points, T_0 or T_1**
0.01	1.4458	$T_1 = (0.00889, 0.24735, 0.00301, 0.00450, 0.01050, 0.72575)$
0.05	1.3939	$T_1 = (0.04448, 0.24735, 0.00266, 0.00397, 0.01204, 0.69226)$
0.10	1.3290	$T_1 = (0.08897, 0.16671, 0.00301, 0.00450, 0.01051, 0.67474)$
0.40	0.9396	$T_0 = (0.35588, 0.23241, 0, 0, 0, 0.41170)$
0.50	0.8098	$T_0 = (0.44485, 0.20032, 0, 0, 0, 0.35483)$

Table 2 shows that increasing the value of c gives a consequence of reducing the value of $\mathcal{R}_0$. When $\mathcal{R}_0 > 1$, the equilibrium point T_1 exists and becomes stable, but when $\mathcal{R}_0 < 1$, the equilibrium point T_1 does not exist and the equilibrium point T_0 becomes stable. The exposed, infected, and carriers will become extinct and the hepatitis B virus will disappears from the population.

5.2. Effect of vaccination to the value of $\mathcal{R}_0$ and the equilibrium points

In this simulation, we suppose that the values of parameter of model (2) are given as $c = 0.015$, $\beta = 0.0121$, $\rho = 7.5$, $\psi = 0.0015$, $\lambda_1 = 6$, $\lambda_2 = 4$, $\lambda_3 = 0.025$, $\lambda_4 = 0.05$, $P = 0.08$, and $u_2 = 0.1$ in appropriate units. There are five values of u_1 used in this simulation, namely $u_1 = 0.07$, $u_1 = 0.10$, $u_1 = 0.15$, $u_1 = 0.20$, and $u_1 = 0.30$. The effect of increasing value of u_1 to the value of $\mathcal{R}_0$ and equilibrium point is given in table 3.

Table 3. The effect of vaccines on susceptible individual to the spread of hepatitis B virus.

Parameter u_1	**$\mathcal{R}_0$**	**Equilibrium points, T_0 or T_1**
0.07	1.8751	$T_1 = (0.01335, 0.24735, 0.00453, 0.00678, 0.01581, 0.71218)$
0.10	1.5281	$T_1 = (0.01335, 0.24735, 0.00336, 0.00502, 0.01171, 0.71922)$
0.15	1.1678	$T_1 = (0.01335, 0.24735, 0.00139, 0.00209, 0.00487, 0.73095)$
0.20	0.9451	$T_0 = (0.01334559, 0.23377047, 0, 0, 0, 0.75288395)$
0.30	0.6841	$T_0 = (0.01334559, 0.16921082, 0, 0, 0, 0.81744389)$

Table 3 shows that increasing the value of u_1 gives a consequence of reducing the value of $\mathcal{R}_0$. When $\mathcal{R}_0 > 1$, the equilibrium point T_1 exists and becomes stable, but when $\mathcal{R}_0 < 1$, the equilibrium point T_1 does not exist and the equilibrium point T_0 becomes stable. The exposed, infected, and carriers will become extinct and the hepatitis B virus will disappears from the population.

5.3. Effect of treatment to the value of $\mathcal{R}_0$ and the equilibrium points

In this simulation, we suppose that the values of parameter of model (2) are given as $c = 0.015$, $\beta = 0.0121$, $\rho = 7.5$, $\psi = 0.0015$, $\lambda_1 = 6$, $\lambda_2 = 4$, $\lambda_3 = 0.025$, $\lambda_4 = 0.05$, $P = 0.08$, and $u_1 = 0.11$ in appropriate units. There are five values of u_2 used in this simulation, namely $u_2 = 0.02$, $u_2 = 0.05$,

$u_2 = 0.10$, $u_2 = 0.30$, and $u_2 = 0.50$. The effect of increasing value of u_2 to the value of $\mathcal{R}_0$ and equilibrium point is given in table 4.

Table 4. The effect of treatment on infected to the spread of hepatitis B virus.

Parameter u_2	$\mathcal{R}_0$	Equilibrium points, T_0 or T_1
0.02	2.5253	$T_1 = (0.01335, 0.14098, 0.00565, 0.00844, 0.04731, 0.78428)$
0.05	1.8843	$T_1 = (0.01335, 0.18894, 0.00448, 0.00671, 0.02464, 0.76188)$
0.10	1.4393	$T_1 = (0.01335, 0.24735, 0.00296, 0.00443, 0.01034, 0.72156)$
0.30	0.9794	$T_0 = (0.01334559, 0.35602114, 0, 0, 0, 0.63063327)$
0.50	0.8620	$T_0 = (0.01334558, 0.35602114, 0, 0, 0, 0.63063327)$

Table 4 shows that increasing the value of u_2 gives a consequence of reducing the value of $\mathcal{R}_0$. When $\mathcal{R}_0 > 1$, the equilibrium point T_1 exists and becomes stable, but when $\mathcal{R}_0 < 1$, the equilibrium point T_1 does not exist and the equilibrium point T_0 becomes stable. The exposed, infected, and carriers will become extinct and the hepatitis B virus will disappear from the population.

6. Conclusions

In the model of the spread of hepatitis B virus using the standard SEIR model has been developed by considering vaccination and treatment in compartments. This strategy aims to reduce the rate of spread of hepatitis B virus. The model was classified into 6 compartments which include immunized, exposed, infected, carrier, and recovered compartments and written as an MSEICR model. The model has an endemic and non-endemic equilibrium points. The existence of endemic equilibrium point depends on the value of $\mathcal{R}_0$. When the value of $\mathcal{R}_0 < 1$ then the endemic equilibrium point does not exist and the non-endemic equilibrium point is asymptotically stable. When the value of $\mathcal{R}_0 > 1$ then the endemic equilibrium point exists and it is asymptotically stable.

From the sensitivity analysis it was concluded that vaccination and also treatment give an effect on value of $\mathcal{R}_0$. Increasing the value of the vaccine in the immunized compartment or in the suspected compartment may decrease the value of $\mathcal{R}_0 > 1$ becomes $\mathcal{R}_0 < 1$. This means that giving vaccines may change endemic condition to the non-endemic condition. Treatment in the infected compartment may also affect the value of $\mathcal{R}_0$. Increasing the portion of treatment in the infected compartment may also reduce the value of $\mathcal{R}_0 > 1$ becomes $\mathcal{R}_0 < 1$. This means that the treatment may change endemic conditions to non-endemic condition.

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